My house was lost; does the mortgage company have any right to any of the insurance money?

Probably. Most mortgages in the United States are essentially identical except for the parts specific to your particular house and your particular loan. When your agreed with your bank to borrow money, you probably agreed that you would carry insurance on your house. In fact, you probably couldn't get the loan without providing proof of insurance. The consequence of that agreement is that when your insurance company pays your claim – at least the part of it funding the rebuild or replacement of your home – that check likely will be written payable to both you AND to your mortgage company.

If the insurance check is written to both me and the mortgage company, then which of the two of us get to deposit the money?

The mortgage company. Neither of you can cash the check without the other one's signature. While the law in many (maybe all) places is silent about who then controls the money, that's because the answer is generally assumed to be (and understood to be) that the bank controls the money, since until your home is rebuilt the money is the only collateral left to secure the mortgage. You could fight this, but it will require a lot of time and energy on your part chasing an unlikely victory.

How do I pay my contractor if the mortgage company holds all of the money?

Most of the time the mortgage company releases the money to you in thirds. One third is released when you begin construction. One third is released when you are roughly halfway done (often this is thought of as the home is framed). The last third is released upon substantial completion.

Can the mortgage company simply apply the money to paying off my loan (even if I don't want them to)?

Probably not. Again, most mortgage loan documents read almost the same. In your documents you probably agreed that you must repair or rebuild your home. If the mortgage company applied the insurance money to the loan instead of letting you use it to rebuild, then that would make it harder for you to rebuild. That choice by the mortgage company would expose it to an argument that it was acting in "bad faith." The mortgage company probably does not want to fight that fight. But it might. And then, everyone would have to really study the details of the documents to see what everyone agreed to.

Can I simply apply the money to paying off my loan (even if the mortgage company doesn't want me to)?

Probably. Most mortgage loan documents allow the borrower – at their option – to pay off the loan early, in whole or in part.

While the mortgage company holds the money, does it collect interest?

It should, but often it does not unless you fight about it. Financial companies basically never just put money in a locked drawer; instead, they use that money to try to make money for themselves, doing things like investing it or loaning it to others. But this is your money, and you did not choose to deposit it with this financial company (because, say, they paid a nice rate of interest) – you were forced to deposit it with them. You should get some interest on the money. But it seems that most of the time this does not happen without a lot of wrangling. If your property is in California, then state law helps you a bit because California has a law – section 2954.8 of the Civil Code – that says when the bank holds money related to the property, the money collects simple interest annually of 2%. Even in California, however, many banks fight this and it requires tenacity to get it.

If my insurance checks are more than I have left owing on the mortgage, does the mortgage company still get to keep control of all of the money?

It shouldn't. But again convincing them of this may take some persistence. In talking to your mortgage company about this (should you choose to do so), ask them what is their justification for being "over-collateralized" when doing so puts as risk your ability to rebuild (which your mortgage requires you to do).